



**TEACHING INSTRUCTIONAL DESIGN (BRP)**

**COURSE**

**RELATIVISTIC QUANTUM MECHANICS**

**by**

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Depok  
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## **PREFACE**

The Relativistic Quantum Mechanics course is a 7<sup>th</sup> semester course for students who have finished the Quantum Mechanics 2 (Non-relativistic) course. For 12 weeks students will receive lecture materials on transitioning from non-relativistic quantum mechanics into relativistic quantum mechanics, the Klein-Gordon equation, the covariant formulation of the Maxwell equations, the Dirac equation, and the application of those equations in a high-energy (relativistic) scattering process. In this course, the formulation of Feynman's diagram will also be discussed to help with students' calculations. During the 12 weeks of lectures, students will receive at least 6 assignments to be done at home. After receiving the contents of this course and doing said assignments, students will pick one project topic, which will be written down in a paper and presented before the lecturer on the 13<sup>th</sup> and 14<sup>th</sup> week.

It is hoped that after finishing the Relativistic Quantum Mechanics course students will be able to calculate formulas for relativistic scattering reactions involving fermions and bosons. Generally, this ability will help students from the Theoretical Nuclear and Particle specialization in doing their undergraduate research thesis.

Depok, 8 September 2013

**Prof. Dr. Terry Mart**

## I. General Information

1. Name of Program / Study Level : Physics / Undergraduate
2. Course Name : Relativistic Quantum Mechanics
3. Course Code : SCFI604411
4. Semester : 7
5. Credit : 4 credits
6. Teaching Method(s) : Interactive lecture, discussion, and presentation
7. Prerequisite course(s) : Quantum Mechanics 2
8. Requisite for course(s) : -
9. Integration Between Other Courses : -
10. Lecturer(s) : Prof. Dr. Terry Mart
11. Course Description : This course explains the concepts and formulations for relativistic quantum mechanics and drills enrolled student's abilities in applying those concepts for problems in nuclear and particle physics.

## II. Course Learning Outcome (CLO) and Sub-CLOs

### A. CLO

After completing this course, students will be able to apply the concepts and formulations in relativistic quantum mechanics for problems in nuclear and particle physics, such as relativistic scattering involving fermions and bosons.

### B. Sub-CLO

After the completion of this course, students will be able to:

1. calculate changes in observable values (such as the expectation value for energy) due to time-dependent and time-independent perturbation;
  - a) calculate the energy of a harmonic oscillator using the Dirac operator,  $\hat{a}$  and  $\hat{a}^\dagger$ ,
  - b) calculate changes in the observable due to perturbations from a small time-independent potential,
  - c) calculate changes in the observable due to perturbations from a small time-dependent potential,
  - d) calculate changes in the observable values using ordinary integration techniques or by using the Dirac operator,  $\hat{a}$  and  $\hat{a}^\dagger$ ,
  - e) write down the characteristics of the Dirac-delta function mathematically.
2. write down and explain the relativistic notation;
  - a) write down the four-vector  $A^\mu$  and explain the difference between it and the three-vector  $\mathbf{A}$ ,
  - b) reduce the four Maxwell equations well known in electromagnetic fields into one covariant Maxwell equation,
  - c) write down and explain the photon wavefunction,
  - d) use minimal substitution to derive the dynamic equation of an electron in a magnetic field,
  - e) explain the Mandelstam variables,
  - f) use the Mandelstam variables for processes with crossing symmetry.
3. explain and use the Klein-Gordon equation;
  - a) derive and use the Klein-Gordon equation,
  - b) calculate the current density and probability from the Klein-Gordon equation,
  - c) explain particle interaction with electromagnetic fields by using minimal substitution on the Klein-Gordon equation,
  - d) calculate the Coulomb scattering amplitude of two spinless point particles with a charge distribution  $A^\mu$  using the Klein-Gordon equation,
  - e) calculate the Coulomb scattering amplitude of two spinless point particles using the Klein-Gordon equation,

- f) calculate the Compton scattering for spinless point particles,
  - g) derive the Feynman rules for the Coulomb scattering of spinless particles,
  - h) calculate the decay rate of a process.
4. explain and use the Dirac equation;
- a) explain the derivation for the Dirac equation and its components,
  - b) explain and derive the characteristics of the Dirac matrices  $\gamma^\mu$ ,
  - c) calculate the equations that are derived from the Dirac matrices  $\gamma^\mu$ ,
  - d) calculate the current density and probability from the Dirac equation,
  - e) derive the Dirac solution for a free particle (zero potential),
  - f) explain the Dirac and Feynman interpretation of negative energy,
  - g) derive the completeness relation,
  - h) derive the spin projection operator,
  - i) use the Dirac equation to calculate the Coulomb scattering amplitude for particles with spin,
  - j) derive the relations in trace algebra,
  - k) derive the formula for the polarization observables,
  - l) calculate the amplitude and cross-section for electron and positron Coulomb scattering,
  - m) calculate the amplitude and cross-section for electron-muon and electron-electron Coulomb scattering,
  - n) derive the amplitude for electron-positron annihilation and to calculate its cross-section,
  - o) derive the Compton scattering amplitude for particles with spin,
  - p) derive the Feynman rules for scattering that involves particles with spin.

### III. Teaching Plan

Week	Sub-CLO	Study Materials	Teaching Method	Time Required	Learning Experiences (*O-E-F)	Sub-CLO Weight on Course (%)	Sub-CLO Achievement Indicator	References
1	1	Calculating the energy of a harmonic oscillator using the Dirac operator, $\hat{a}$ and $\hat{a}^\dagger$	Lecturing	2x50 minutes	70% O, 20% E, 10% F	6		
2	2	<ol style="list-style-type: none"> <li>1. Writing down the four-vector <math>A^\mu</math> and explaining the difference between it and the three-vector <math>\mathbf{A}</math>.</li> <li>2. Reducing the four Maxwell equations well known in electromagnetic fields into one covariant Maxwell equation.</li> </ol>	Lecturing	2x50 minutes	70% O, 20% E, 10% F	6		
3	3	<ol style="list-style-type: none"> <li>1. Writing down and explaining the photon wave function.</li> <li>2. Using minimal substitution to derive the dynamic equation of an electron in a magnetic field.</li> </ol>	Lecturing	2x50 minutes	70% O, 20% E, 10% F	6		
4	4	<ol style="list-style-type: none"> <li>1. Explaining the Mandelstam variables.</li> <li>2. Using the Mandelstam variables for processes with crossing symmetry.</li> </ol>	Lecturing	2x50 minutes	70% O, 20% E, 10% F	6		

5	5	<ol style="list-style-type: none"> <li>1. Deriving and explaining the Klein-Gordon equation.</li> <li>2. Calculating the current density and probability from the Klein-Gordon equation.</li> </ol>	Lecturing	2x50 minutes	70% O, 20% E, 10% F	6		
6	6	<ol style="list-style-type: none"> <li>1. Explaining particle interaction with electromagnetic fields by using minimal substitution on the Klein-Gordon equation.</li> <li>2. Calculating the Coulomb scattering amplitude of two spinless point particles with a charge distribution <math>A^\mu</math> using the Klein-Gordon equation.</li> </ol>	Lecturing	2x50 minutes	70% O, 20% E, 10% F	10		
7	7	<ol style="list-style-type: none"> <li>1. Continue calculating the Coulomb scattering amplitude of two spinless point particles with a charge distribution <math>A^\mu</math> using the Klein-Gordon equation.</li> <li>2. Calculating the Coulomb scattering amplitude of two spinless point particles using the Klein-Gordon equation.</li> </ol>	Lecturing	2x50 minutes	70% O, 20% E, 10% F	10		
8	<b>Midterm Exam</b>							

9	8	<ol style="list-style-type: none"> <li>1. Calculating the Compton scattering for spinless point particles.</li> <li>2. Deriving the Feynman rules for the Coulomb scattering of spinless particles.</li> <li>3. Calculating the decay rate of a process.</li> </ol>	Lecturing	2x50 minutes	70% O, 20% E, 10% F	10		
10	9	<ol style="list-style-type: none"> <li>1. Explaining the derivation for the Dirac equation and its components.</li> <li>2. Explaining and deriving the characteristics of the Dirac matrices <math>\gamma^\mu</math>.</li> </ol>	Lecturing	2x50 minutes	70% O, 20% E, 10% F	10		
11	10	<ol style="list-style-type: none"> <li>1. Continue explaining and deriving the characteristics of the Dirac matrices <math>\gamma^\mu</math>.</li> <li>2. Calculating the equations that are derived from the Dirac matrices <math>\gamma^\mu</math>.</li> </ol>	Lecturing	2x50 minutes	70% O, 20% E, 10% F	10		
12	11	<ol style="list-style-type: none"> <li>1. Calculating the current density and probability from the Dirac equation.</li> <li>2. Deriving the Dirac solution for a free</li> </ol>	Lecturing	2x50 minutes	70% O, 20% E, 10% F	6		



		<p>particle (zero potential).</p> <ol style="list-style-type: none"> <li>Explaining the Dirac and Feynman interpretation of negative energy.</li> <li>Deriving the completeness relation.</li> </ol>						
13	12	<ol style="list-style-type: none"> <li>Deriving the spin projection operator.</li> <li>Using the Dirac equation to calculate the Coulomb scattering amplitude for particles with spin</li> </ol>	Lecturing	2x50 minutes	70% O, 20% E, 10% F	4		
14	13	<ol style="list-style-type: none"> <li>Deriving the relations in trace algebra.</li> <li>Deriving the formula for the polarization observables.</li> <li>Calculating the amplitude and cross-section for electron and positron Coulomb scattering.</li> <li>Calculating the amplitude and cross-section for electron-muon and electron-electron Coulomb scattering.</li> </ol>	Lecturing	2x50 minutes	70% O, 20% E, 10% F	6		

15	14	1. Deriving the amplitude for electron-positron annihilation and calculating its cross-section. 2. Deriving the Compton scattering amplitude for particles with spin.	Lecturing	2x50 minutes	70% O, 20% E, 10% F	4		
16	<b>Final Exam</b>							

\*) O : Orientation  
 E : Exercise  
 F : Feedback

References:

1. F. Halzen and A. D. Martin, *Quarks & Leptons*, John Wiley & Sons, 1984.
2. J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics*, Mc Graw-Hill, 1964.
3. I. J. R. Aitchison, *Relativistic Quantum Mechanics*, Macmillan, 1982.
4. F. Gross, *Relativistic Quantum Mechanics and Field Theory*, John Wiley & Sons, 1993.

#### IV. Assignment Design

Week	Assignment Name	Sub-CLO	Assignment	Scope	Working Procedure	Deadline	Outcome
1	Calculating changes in observable values (such as the expectation value for energy) due to time-dependent and time-independent perturbation	1	Group Assignment	<ol style="list-style-type: none"> <li>1. Calculating the energy of a harmonic oscillator using the Dirac operator, <math>\hat{a}</math> and <math>\hat{a}^\dagger</math>.</li> <li>2. Calculating changes in the observable due to perturbations from a small time-independent potential.</li> <li>3. Calculating changes in the observable due to perturbations from a small time-dependent potential.</li> <li>4. Calculating changes in the observable values using ordinary integration techniques or by using the Dirac operator, <math>\hat{a}</math> and <math>\hat{a}^\dagger</math>.</li> <li>5. Writing down the characteristics of the Dirac-delta function mathematically.</li> </ol>	In-class and online; in groups and independently	2x50 minutes	
2	Writing down and explaining the relativistic notation	2	Group Assignment	<ol style="list-style-type: none"> <li>1. Writing down the four-vector <math>A^\mu</math> and explain the difference between it and the three-vector <math>\mathbf{A}</math>.</li> <li>2. Reducing the four Maxwell equations well known in electromagnetic fields into one covariant Maxwell equation.</li> <li>3. Writing down and explaining the photon wave function.</li> <li>4. Using minimal substitution to derive the dynamic equation of an electron in a magnetic field.</li> <li>5. Explaining the Mandelstam variables.</li> <li>6. Using the Mandelstam variables for processes with crossing symmetry.</li> </ol>	In-class and online; in groups and independently	2x50 minutes	

3	Explaining and using the Klein-Gordon equation	3	Group Assignment	<ol style="list-style-type: none"> <li>1. Deriving and use the Klein-Gordon equation.</li> <li>2. Calculating the current density and probability from the Klein-Gordon equation.</li> <li>3. Explaining particle interaction with electromagnetic fields by using minimal substitution on the Klein-Gordon equation.</li> <li>4. Calculating the Coulomb scattering amplitude of two spinless point particles with a charge distribution <math>A^\mu</math> using the Klein-Gordon equation.</li> <li>5. Calculating the Coulomb scattering amplitude of two spinless point particles using the Klein-Gordon equation.</li> <li>6. Calculating the Compton scattering for spinless point particles.</li> <li>7. Deriving the Feynman rules for the Coulomb scattering of spinless particles.</li> <li>8. Calculating the decay rate of a process.</li> </ol>	In-class and online; in groups and independently	2x50 minutes	
4	Explain and use the Dirac equation	4	Group Assignment	<ol style="list-style-type: none"> <li>1. Explaining the derivation for the Dirac equation and its components.</li> <li>2. Explaining and deriving the characteristics of the Dirac matrices <math>\gamma^\mu</math>.</li> <li>3. Calculating the equations that are derived from the Dirac matrices <math>\gamma^\mu</math>.</li> <li>4. Calculating the current density and probability from the Dirac equation.</li> <li>5. Deriving the Dirac solution for a free particle (zero potential).</li> </ol>	In-class and online; in groups and independently	2x50 minutes	

				<ol style="list-style-type: none"> <li>6. Explaining the Dirac and Feynman interpretation of negative energy.</li> <li>7. Deriving the completeness relation.</li> <li>8. Deriving the spin projection operator.</li> <li>9. Using the Dirac equation to calculate the Coulomb scattering amplitude for particles with spin.</li> <li>10. Deriving the relations in trace algebra.</li> <li>11. Deriving the formula for the polarization observables.</li> <li>12. Calculating the amplitude and cross-section for electron and positron Coulomb scattering.</li> <li>13. Calculating the amplitude and cross-section for electron-muon and electron-electron Coulomb scattering.</li> <li>14. Deriving the amplitude for electron-positron annihilation and calculating its cross-section.</li> <li>15. Deriving the Compton scattering amplitude for particles with spin.</li> <li>16. Deriving the Feynman rules for scattering that involves particles with spin.</li> </ol>			
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## V. Assessment Criteria (Learning Outcome Evaluation)

Evaluation Type	Sub-CLO	Assessment Type	Frequency	Evaluation Weight (%)
Individual Assignment	1-4	Answer sheet	0	20
Group Assignment	5-6	Presentation	14	30
Midterm Exam	1-3	Answer sheet	1	30
Final Exam	4-6	Answer sheet	1	40
<b>Total</b>				<b>100</b>

## VI. Rubric

### A. Criteria of Presentation Score

Score	Presentation Delivery
85-90	Group is able to deliver the explanation logically, fluently, and punctual and be able to answer the questions from other students and lecturer
75-84	Group is able to deliver the explanation logically and fluently and be able to answer the questions from other students and lecturer, but be less punctual on delivering the explanation
65-74	Group is able to deliver the explanation fluently, but be less able to deliver the reasoning logic of the explanation
55-64	Group is less able to deliver the explanation fluently and punctual and be less able to deliver the reasoning logic of the explanation
<55	

### B. Criteria of Assignment and Exam Scores

Score	Answer Quality
100	Answer is very precise and all the concept and main component are explained completely
76-99	Answer is fairly precise and the concept and main component are explained fairly complete
51-75	Answer is less precise and the concept and main component are explained less complete
26-50	Answer is poorly precise and the concept and main component are explained poorly complete
<25	Answer is wrong