



TEACHING INSTRUCTIONAL DESIGN (BRP)
COURSE
ANGULAR MOMENTUM THEORY

by

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PREFACE

The Angular Momentum Theory course is a Nuclear and Particle Physics specialization course for 7th term students who have completed the Quantum Mechanics 2 course. For 14 weeks of lectures, students will receive lecture materials on the definition of angular momentum, commutative properties, and eigenvalue commutators, the summation of two angular momenta, the definition of the Clebsch-Gordan coefficient, the 3-j, 6-j, and 9-j symbols, rotation operators and their orthogonal properties, spherical harmonic functions, irreducible tensors, the Wigner-Eckart theorem, Racah coefficients, Maxwell's equations and multipole fields in spherical coordinate, static interactions and spin $\frac{1}{2}$ interactions, and applications in physics. During the course, students will be given 10 assignments containing problem sets that have to be done individually (homework).

It is hoped that after completing the Angular Momentum Theory course, students will be able to understand angular momentum theory, the summation of angular momenta, the Clebsch-Gordan coefficient, the 3-j, 6-j, and 9-j symbols, and their applications within physics. The skills gained in the Angular Momentum Theory course will help students from the Theoretical Nuclear and Particle specialization in doing their research thesis, especially if their thesis topic relates to nuclear, particle, and few-body physics.

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Prof. Dr. Terry Mart

I. General Information

1. Name of Program / Study Level : Physics / Undergraduate
2. Course Name : Angular Momentum Theory
3. Course Code : SCFI602114
4. Semester : 7
5. Credit : 4 credits
6. Teaching Method(s) : Lecturing, individual assignment, written exam
7. Prerequisite course(s) : Quantum Mechanics 2
8. Requisite for course(s) : -
9. Integration Between Other Courses : -
10. Lecturer(s) : Prof. Dr. Terry Mart
11. Course Description : The Angular Momentum Theory course teaches students about formulating few-body angular momenta and its application within physics. The topics discussed include the definition of angular momentum, commutative properties, and eigenvalue commutators, the summation of two angular momenta, the definition of the Clebsch-Gordan coefficient, relations for the Clebsch-Gordan coefficient, calculation of the Clebsch-Gordan coefficient, the 3-j, 6-j, and 9-j symbols, rotation operators and their orthogonal properties, spherical harmonic functions, irreducible tensors, the Wigner-Eckart theorem, Racah coefficients, Maxwell's equations and multipole fields in spherical coordinate, static interactions and spin- $\frac{1}{2}$ interactions, and applications for nuclear systems and alpha particle emission by the nucleus.

II. Course Learning Outcome (CLO) and Sub-CLOs

A. CLO

After the completion of this course, students will be able to derive relations in angular momentum theory and apply them in problem within physics that includes quantum mechanics, nuclear physics, particle physics, and few-body physics. (PLO(s) 1, 2, 5, 6, 7)

B. Sub-CLO

It is hoped that after the completion of this course students are able to:

1. derive the fundamental principles used in angular momentum theory (C3),
2. derive and calculate angular momentum operators (C3),
3. derive the coupling of two angular momenta (C3),
4. calculate and test transformation characteristics due to rotation (C3, C4),
5. derive and apply the Wigner-Eckart theorem (C3),
6. derive Racah coefficients and demonstrate their basic application (C3),
7. derive Maxwell's equations in spherical coordinates (C3),
8. apply formulations in angular momentum theory in solving problems within physics (C3).

III. Teaching Plan

| Week | Sub-CLO | Study Materials | Teaching Method | Time Required | Learning Experiences (*O-E-F) | Sub-CLO Weight on Course (%) | Sub-CLO Achievement Indicator | References |
|------|---------------------|--|-----------------|---------------|-------------------------------|------------------------------|---|-------------------------------------|
| 1 | 1 | Operators and unitary transformations | Lecturing | 200 minutes | 60% O, 20% E, 20% F | 5 | Students can derive the fundamental principles used in angular momentum theory. | Ref. 1 p. 3-9; Ref. 2 p. 13 |
| 2 | 1 | Diagonalization and exponential form of operators, definition of angular momentum | Lecturing | 200 minutes | 60% O, 20% E, 20% F | 5 | Students can derive and calculate angular momentum operators. | Ref. 1 p. 10-14, Ref. 2 p. 10-30 |
| 3 | 1 | Commutative properties and eigenvalue of commutators, physical interpretation of angular momentum | Lecturing | 200 minutes | 60% O, 20% E, 20% F | 7 | Students can derive and calculate angular momentum operators. | Ref. 1 p. 20-31, Ref. 2 p. 18-26 |
| 4 | 2 | Summation of two angular momenta, definition of the Clebsch-Gordan coefficient | Lecturing | 200 minutes | 60% O, 20% E, 20% F | 8 | Students can derive the coupling of two angular momenta. | Ref. 1 p. 32-35; Ref. 2 p. 31-34 |
| 5 | 2 | Relations related to the Clebsch-Gordan coefficient, calculation of the Clebsch-Gordan coefficient | Lecturing | 200 minutes | 60% O, 20% E, 20% F | 8 | Students can derive the coupling of two angular momenta. | Ref. 1 p. 37-47; Ref. 2 p. 37-43 |
| 6 | 3 | Matrix representation of rotation operators | Lecturing | 200 minutes | 60% O, 20% E, 20% F | 5 | Students can calculate and test transformation characteristics due to rotation. | Ref. 1 p. 48-56 |
| 7 | 3 | Rotation operator and its orthogonality | Lecturing | 200 minutes | 60% O, 20% E, 20% F | 7 | Students can calculate and test transformation characteristics due to rotation. | Ref. 1 p. 62-75 |
| 8 | Midterm Exam | | | | | | | |

| | | | | | | | | |
|----|-------------------|---|-----------|-------------|---------------------|----|---|--|
| 9 | 4 | Spherical harmonic function, irreducible tensors | Lecturing | 200 minutes | 60% O, 20% E, 20% F | 8 | Students can derive and apply the Wigner-Eckart theorem. | Ref. 1 p. 76-81; Ref. 2 p. 9, 124 |
| 10 | 4 | Wigner-Eckart theorem, the 3-j, 6-j, and 9-j symbols | Lecturing | 200 minutes | 60% O, 20% E, 20% F | 10 | Students can derive and apply the Wigner-Eckart theorem. | Ref. 1 p. 85-93; Ref. 2 p. 4, 45-50, 92-100, 100-107 |
| 11 | 4 | Summation of three angular momenta, Racah coefficient | Lecturing | 200 minutes | 60% O, 20% E, 20% F | 10 | Students can derive Racah coefficients and demonstrate their basic application | Ref. 1 p. 107-126; Ref. 2 p. 52, 97 |
| 12 | 5 | Maxwell's equations and multipole fields in spherical coordinates | Lecturing | 200 minutes | 60% O, 20% E, 20% F | 7 | Students can derive Maxwell's equations in spherical coordinates. | Ref. 1 p. 127-139 |
| 13 | 5 | Static interactions and interactions involving particles with spin- $1/2$ | Lecturing | 200 minutes | 60% O, 20% E, 20% F | 7 | Students can apply formulations in angular momentum theory in solving problems within physics | Ref. 1 p. 140-149 |
| 14 | 6 | Applications in nuclear reactions | Lecturing | 200 minutes | 60% O, 20% E, 20% F | 7 | Students can apply formulations in angular momentum theory in solving problems within physics | Ref. 1 p. 187-203 |
| 15 | 6 | Applications in the emission of α particles by the nucleus | Lecturing | 200 minutes | 60% O, 20% E, 20% F | 6 | Students can apply formulations in angular momentum theory in solving problems within physics | Ref. 1 p. 161-186 |
| 16 | Final Exam | | | | | | | |

*) O : Orientation
E : Experience
F : Feedback

References:

1. M. E. Rose, *Elementary Theory of Angular Momentum*, Dover Books on Physics, reprint edition, 2011.
2. R. Edmonds, *Angular Momentum in Quantum Mechanics*, Princeton University Press, Reissue edition, 1996.

IV. Assignment Design

| Week | Assignment Name | Sub-CLO | Assignment | Scope | Working Procedure | Deadline | Outcome |
|------|--------------------------|---------|-------------|---|---------------------|----------|--------------|
| 2 | Individual Assignment 1 | 1 | Problem set | Operators and unitary transformations, diagonalization and exponential form of operators, definition of angular momentum | Individual homework | 1 week | Answer sheet |
| 3 | Individual Assignment 2 | 1 | Problem set | Commutative properties and eigenvalue of commutators, physical interpretation of angular momentum | Individual homework | 1 week | Answer sheet |
| 4 | Individual Assignment 3 | 2 | Problem set | Summation of two angular momenta, definition of the Clebsch-Gordan coefficient | Individual homework | 1 week | Answer sheet |
| 5 | Individual Assignment 4 | 2 | Problem set | Relations related to the Clebsch-Gordan coefficient, calculation of the Clebsch-Gordan coefficient | Individual homework | 1 week | Answer sheet |
| 7 | Individual Assignment 5 | 3 | Problem set | Matrix representation of rotation operators, rotation operator, and its orthogonality | Individual homework | 1 week | Answer sheet |
| 10 | Individual Assignment 6 | 4 | Problem set | Spherical harmonic function, irreducible tensors | Individual homework | 1 week | Answer sheet |
| 11 | Individual Assignment 7 | 4 | Problem set | Wigner-Eckart theorem, the 3-j, 6-j, and 9-j symbols | Individual homework | 1 week | Answer sheet |
| 12 | Individual Assignment 8 | 4, 5 | Problem set | Summation of three angular momenta, Racah coefficient | Individual homework | 1 week | Answer sheet |
| 13 | Individual Assignment 9 | 5 | Problem set | Maxwell's equations and multipole fields in spherical coordinates, static interactions, and interactions involving particles with spin- $1/2$ | Individual homework | 1 week | Answer sheet |
| 15 | Individual Assignment 10 | 6 | Problem set | Applications in nuclear reactions and the emission of α particles by the nucleus | Individual homework | 1 week | Answer sheet |

V. Assessment Criteria (Learning Outcome Evaluation)

| Evaluation Type | Sub-CLO | Assessment Type | Frequency | Evaluation Weight (%) |
|------------------------|----------------|------------------------|------------------|------------------------------|
| Individual Assignment | 1-6 | Answer sheet | 10 | 30 |
| Midterm Exam | 1-3 | Answer sheet | 1 | 30 |
| Final Exam | 4-6 | Answer sheet | 1 | 40 |
| Total | | | | 100 |

VI. Rubric

A. Criteria of Assignment and Exam Scores

| Score | Answer Quality |
|--------------|---|
| 100 | Answer is very precise and all the concept and main component are explained completely |
| 76-99 | Answer is fairly precise and the concept and main component are explained fairly complete |
| 51-75 | Answer is less precise and the concept and main component are explained less complete |
| 26-50 | Answer is poorly precise and the concept and main component are explained poorly complete |
| <25 | Answer is wrong |