Solid-state Physics

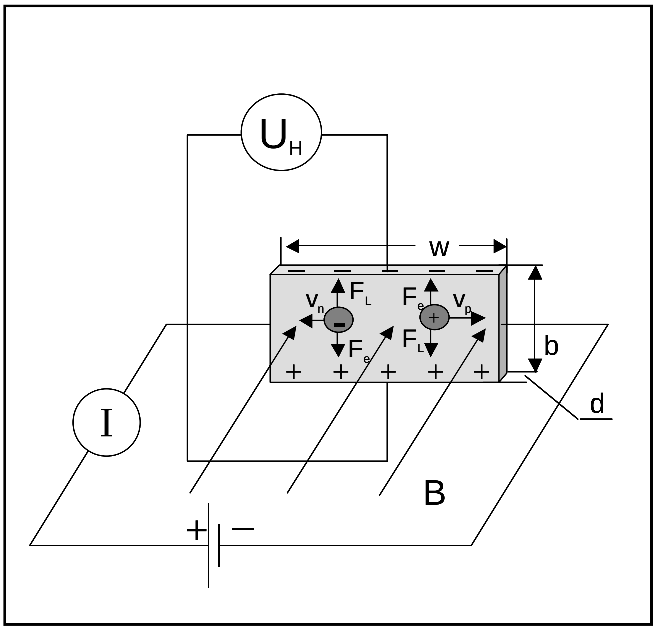
Conduction phenomena
Hall Effect

Determining the density and mobility of charge carriers in n-germanium

Objects of the experiment

- Measuring of the Hall voltage as function of the current at a constant magnetic field: determination of the density and mobility of charge carriers.
- Measuring of the Hall voltage for as function of the magnetic field at a constant current:: determination of the Hall coefficient.
- Measuring of the Hall voltage as function of temperature: investigation of the transition from extrinsic to intrinsic conductivity.

Fig. 1: Hall effect in a rectangular sample of thickness d, height b and length w: At equilibrium conditions the Lorentz force F_L acting on the moving charge carriers is balanced by the electrical force F_e which is due to the electric field of the Hall effect.



Principles

The Hall effect is an important experimental method of investigation to determine the microscopic parameters of the charge transport in metals or doped semiconductors.

To investigate the Hall effect in this experiment a rectangular strip of n-doped germanium is placed in a uniform magnetic field B according Fig. 1. If a current I flows through the rectangular shaped sample an electrical voltage (Hall voltage) is set up perpendicular to the magnetic field B and the current I due to the Hall effect:

$$U_H = R_H \cdot \frac{I \cdot B}{d} \qquad \text{(I)}$$

R_H is the Hall coefficient which depends on the material and the temperature. At equilibrium conditions (Fig. 1) for weak magnetic fields the Hall coefficient R_H can be expressed as function of the charge density (carrier concentration) and the mobility of electrons and holes:

$$R_H = \frac{1}{e_0} \cdot \frac{p \cdot \mu_p^2 - n \cdot \mu_n^2}{(p \cdot \mu_p + n \cdot \mu_n)^2} \qquad \text{(II)}$$

$e_0 = 1.602 \cdot 10^{-19}$ As (elementary charge)

$n = n_E + n_S$ (total density of electrons)

n_S: density of electrons (electron conduction due to n-doping)

$p = p_E$: density of holes (intrinsic conduction)

p_E: density of holes (intrinsic conduction)

μ_p: mobility of holes

μ_n mobility of electrons

From equation (II) follows: The polarity of predominant charge carriers can be determined from the Hall coefficient R_H if the directions of the current I and magnetic field B are known. The thinner the conducting strip the higher the Hall voltage.

The doping of group V elements like e.g. As, P or Sb into the crystal lattice of germanium creates additional electrons in the conduction band (Fig. 2).

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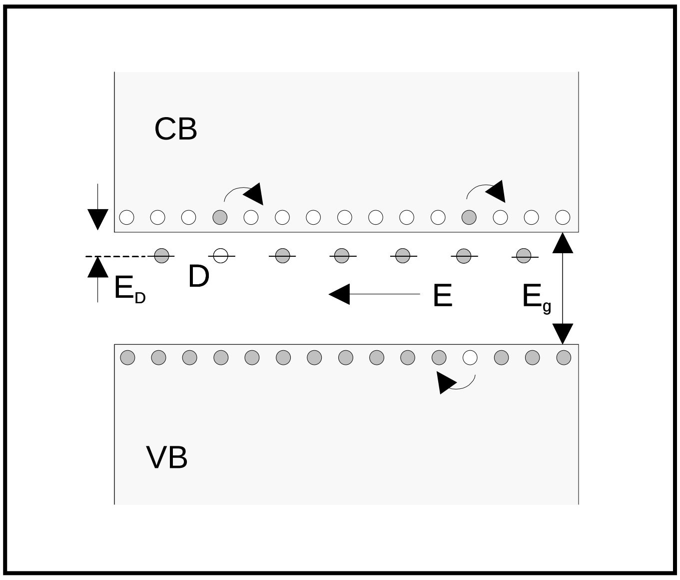

Fig. 2: Simplified diagram of extrinsic (left) and intrinsic conduction (right) under influence of an electric field E: Incorporating of dopants (donors D) into the crystal lattice creates negative charge carriers (electrons) in the conduction band (CB). With increasing temperature the thermal energy of valence electrons increases allowing them to breach the energy gap E_g into the conduction band (CB) leaving a vacancy called hole in the VB.

Their activation energy E_D of about 0.01 eV is significantly smaller than the activation energy E_g (band gap) to generate electrons and holes by thermal activation (intrinsic charge carriers). At room temperatures in n-doped germanium the density of electrons n_S can predominate the density of intrinsic charge carriers (n_E and p_E). In this case where the charge transport is predominately due to electrons from the dopants ($n = n_E = p_E \approx 0$). The density of n_S can be determined by measuring the Hall voltage U_H as function of the current I. With equation (I) and (II) follows:

$$n_S = \frac{B}{e_0 \cdot d} \cdot \frac{I}{U_H} \qquad \text{(III)}$$

The mobility is a measure of the interaction between the charge carriers and the crystal lattice. The mobility is defined as (in case n-doped germanium it is the mobility μ_n of the electrons created by the dopants, i.e. donators):

$$\mu_n = \frac{v_p}{E} \qquad \text{(IV)}$$

v_n: drift velocity

E: electric field due to the voltage drop

The electric filed E can be determined by the voltage drop U and the length w of the n-doped germanium strip:

$$E = \frac{U}{w} \qquad \text{(V)}$$

The drift velocity v_n can be determined from the equilibrium condition, where the Lorentz force compensates the electrical force which is due to the Hall field (Fig. 1)

$$e_0 \cdot v_d \cdot B = e_0 \cdot E_H \qquad \text{(VI)}$$

which can be expressed using the relation $E_H = b \cdot U_H$ as

$$v_d = \frac{U_H}{b \cdot B} \qquad \text{(VII)}$$

Substituting equation (V) and (VII) in equation (IV) the mobility μ_n of holes can be estimated at room temperatures as follows:

$$\mu_n = \frac{U_H \cdot w}{b \cdot B \cdot U} \qquad \text{(VIII)}$$

The current I in a semiconductor crystal is made up of both hole currents and electron currents (Fig. 1):

$$I = b \cdot d\,(n_p \cdot \mu_p + n_n \cdot \mu_n) \qquad \text{(IX)}$$

The carrier density depends on the dopant concentration and the temperature. Three different regions can be distinguished for n-doped germanium: At very low temperatures the excitation from electrons of the donator levels into the conduction band is the only source of charge carriers. The density of “dopant electrons” n_S increases with temperature. It follows a region where the density n_S is independent of temperature as all donator levels are unoccupied (extrinsic conductivity). In this regime the charge transport due to intrinsic charge carriers can be neglected. A further increase in temperature leads to a direct thermal excitation of electrons from the valence band into the conduction band. The charge transport increases due to intrinsic conductivity and finally predominates (Fig. 2). These transition from pure extrinsic conduction to a predominately intrinsic conduction can be observed by measuring the Hall voltage U_H as function of the temperature.

To describe the Hall voltage as function of temperature U_H based on a simple theory equation (I) and (II) have to be extended in the following way:

It is assumed that the mobility of electrons and holes are different. Introducing the ratio of the mobility

$$k = \frac{\mu_n}{\mu_p} \qquad \text{(X)}$$

equation (II) can be rewritten as follows:

$$R_H = \frac{1}{e_0} \cdot \frac{p - n \cdot k^2}{(p + n \cdot k)^2} \qquad \text{(XI)}$$

For undoped semiconductors the temperature dependency of the charge carriers can be assumed as

$$n = n_0 \cdot e^{-\frac{E_g}{2 \cdot k_B \cdot T}}$$

$$p = p_0 \cdot e^{-\frac{E_g}{2 \cdot k_B \cdot T}} \qquad \text{(XII)}$$

$k_B = 1.36\ 10^{-23}$ J/K: Boltzmann constant

The product of the densities n and p is temperature dependent:

$$n \cdot p = (n_E + n_S) \cdot p_E = \eta^2 \qquad \text{(XIII)}$$

where the effective state density η is approximated as

$$\eta^2 = N_0 \cdot e^{-\frac{E_g}{k_B \cdot T}} \qquad \text{(XIV)}$$

In the extrinsic conductivity regime the density n_S can be determined according equation (III). For the intrinsic charge carriers $p_E = n_E$ which leads to a quadratic equation for n_E with the solution:

$$n_E = -\frac{n_S}{2} + \sqrt{\frac{n_S^2}{4} + \eta^2} \qquad \text{(XV)}$$

With equations (XI) and (XV) together with the relations $n = n_E + n_S$ and $p = p_E$ the temperature dependency of Hall voltage U_H can be simulated. Using for E_g = 0.7 eV the result of experiment P7.2.1.5 as estimate value for the simulation only two unknown parameters N_0 and k are left.

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Apparatus

1 Base unit for Hall effect Ge................................ 586 850
1 n-doped Ge plug-in board................................. 586 853

1 Combi B-Sensor S.. 524 0381
1 Extension cable, 15-pole 501 11
1 Sensor CASSY... 524 010
1 CASSY Lab 524 200
2 AC/DC Power supply 0 to 15 V, 5 A 521 501
1 DC Power Supply 0...16 V, 0...5 A..................... 521 545
1 DC power supply 521 541

1 U-core with yoke.. 562 11
1 Pair of bored pole pieces.................................. 560 31
2 Coil with 250 turns.. 562 13
1 Stand rod, 25 cm 300 41
1 Leybold Multi clamp.. 301 01
1 Stand base, V-shape, 20 cm 300 02
7 Pair of cables, 1 m, red and blue...................... 501 46

additionally required:

PC with Windows 95/98/NT or higher

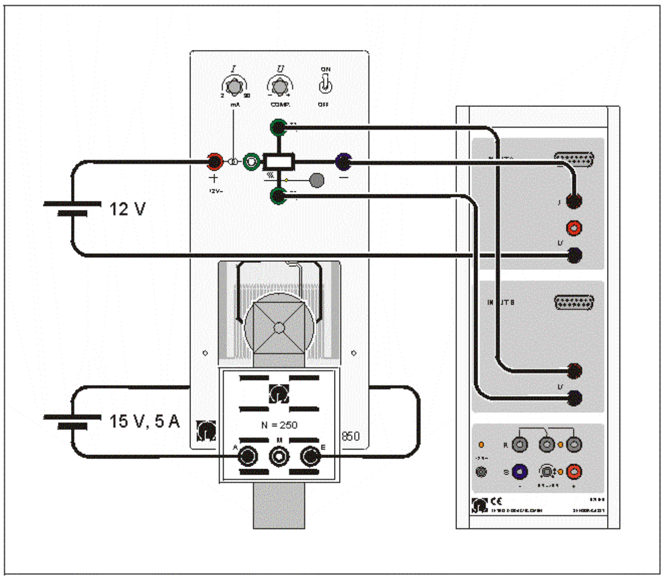

Fig. 2: Experimental setup (wiring diagram) for measuring the Hall voltage as function of the current I.

Setup

Mounting and connecting the plug-in board:

- Insert the plug-in board with the n-doped Ge crystal into the DIN socket on the base unit for Hall effect until the pins engage in the holes.
- Carefully insert the plug-in board with DIN plug into the DIN socket on Insert the base unit with rod into the hole of the U-core all the way to the stop; make sure that the plug-in board is seated parallel to the U-core (see instruction sheet base unit Hall effect 586 850).
- Carefully attach the pair of bored pole pieces with additional pole piece, and slide the additional pole piece as far as the spacers of the plug-in boards (make sure that the plug-in board is not bent).
- Turn the current limiter of the current-controlled power supply to the left stop, and connect the power supply.

Safety notes

The n-doped Ge crystal is extremely fragile:

- Handle the plug-in board carefully and do not subject it to mechanical shocks or loads.

Due to its high specific resistance, the p-doped Ge crystal warms up even if only the cross-current is applied:

- Do not exceed the maximum cross-current I = 33 mA.
- Turn the control knob for the cross-current on the base unit for Hall effect to the left stop.

Measuring the magnetic field:

- The B-probe is fixed by the Stand rod to the V-shaped Stand base.
- Before the measuring the magnetic induction of the field B place the B-probe carefully in the gap (see instruction sheet base unit Hall effect 586 850) after the apparatus is adjusted.
- For the measurement connect B-probe to the Sensor CASSY using the extension cable.

Compensation of the Hall voltage:

- Before performing a measurement with a constant current I the Hall voltage have to be compensated for B = 0 T:
- 1. For measuring the current I connect the cables to the Input A of the Sensor CASSY (Fig. 3, see also instruction sheet base unit Hall effect 586 850).
- 2. For measuring the Hall voltage U_H connect the cables to the Input B of the Sensor CASSY (Fig. 3 see also instruction sheet base unit Hall effect 586 850).
- 3. Set the cross-current I to the maximum value (see instruction manual for n-doped Ge crystal 586 852), switch on the compensation and zero the Hall voltage U_H using the compensation knob.

Measuring the voltage drop:

- For measuring the voltage drop U connect the cables to the Input B of Sensor CASSY (see instruction sheet base unit Hall effect 586 850 measure the conductivity as function of temperature).
- Connect the cables to the Input A of the Sensor CASSY to measure the current I (see instruction sheet base unit Hall effect 586 850).
- Set the current I to the maximum value and measure the voltage drop U.

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Measuring the temperature:

- For measuring the temperature ϑ connect the output signal of the heater to Input A of the Sensor CASSY (see instruction sheet base unit Hall effect 586 850 and Physics Leaflets P7.2.1.5.)

Carrying out the experiment

a) Measuring the Hall voltage as function of current

- First compensate the Hall voltage (see above).
- Set the magnetic field B to a desired value and measure the magnetic flux density B (see above).
- Set the current to the maximum value and measure the voltage drop U.
- Measure the Hall voltage U_H (Input B on Sensor CASSY) as function of the current I (Input A on Sensor CASSY).
- After connecting the cables set the parameters with .
- For measuring use the button or F9 in manual measuring mode.
- Safe your measurement .

b) Measuring the Hall voltage as function of magnetic field

- First compensate the Hall voltage (see above).
- Set the current I to a desired value.
- Measure the Hall voltage U_H (Input B on Sensor CASSY) as function of the magnetic field B (Input A on Sensor CASSY).
- After connecting the cables set the parameters with .
- For measuring use the button or F9 in manual measuring mode.
- Safe your measurement .

c) Measuring the Hall voltage as function of temperature

- First compensate the Hall voltage U_H (see above) and set the current I to a desired value.
- Set the magnetic field B to a desired value (see above).
- Measure the Hall voltage U_H (Input B on Sensor CASSY) as function of the Temperature ϑ (Input A on Sensor CASSY, see above).

Measuring example

a) Measuring the Hall voltage as function of current

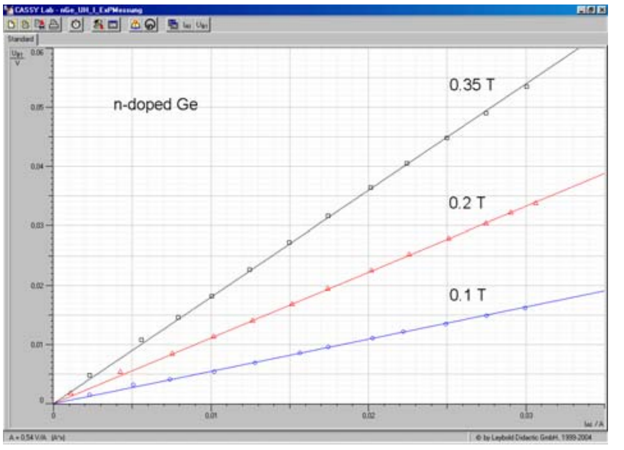

Fig. 4: Hall voltage U_H as function of the current I for different magnetic fields. The straight lines correspond to a fit according equation (I).

current: I = 30 mA

voltage drop: U = 1,1 V.

b) Measuring the Hall voltage as function of magnetic field

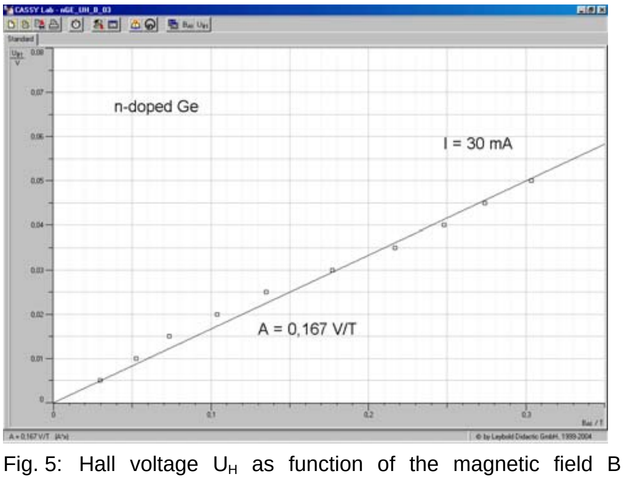

Fig. 5: Hall voltage U_H as function of the magnetic field B for I = 30 mA.. The straight line with slope A corresponds to a fit according equation (I).

c) Measuring the Hall voltage as function of temperature

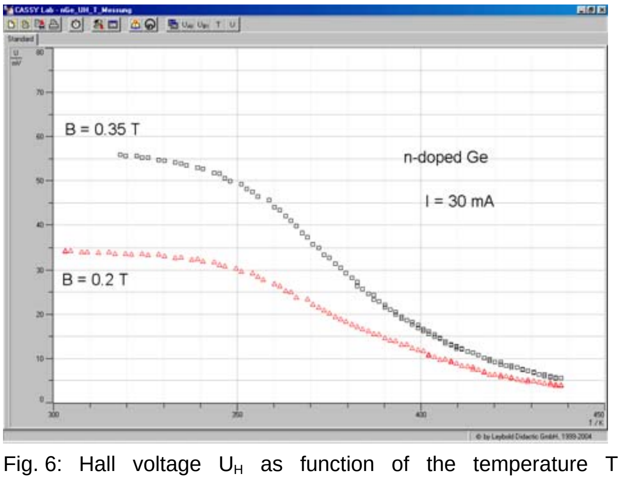

Fig. 6: Hall voltage U_H as function of the temperature T for I = 30 mA and different magnetic fields B.

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Evaluation and results

a) Measuring the Hall voltage as function of current

For the measurement with e. g. B = 0,35 T and I = 30 mA in Fig. 4 the slope

$$A = \frac{R_H \cdot B}{d} = 1.8 \frac{V}{A}$$

is obtained by the fitting a straight line through the origin (right mouse click in the diagram and "fit function"). With the linear regression result and equation (III) the density p_S of holes in the extrinsic conducting regime can determined as follows:

$d = 1{\cdot}10^{-3}$ m

B = 0.35 T

$$n_S = \frac{B}{e_0 \cdot d \cdot A} = 1.2 \cdot 10^{21} \frac{1}{m^3}$$

With the experimental results at room temperature

U = 1.4 V

B = 0.35 T

U_H = 72 mV

and the dimensions of the n-doped germanium strip

b = 10 mm

w = 20 mm

the drift velocity v_n (equation (VII)) and the mobility μ_n (equation (VIII)) of the charge carriers in the extrinsic region can be estimated:

$$v_d = \frac{U_H}{b \cdot B} = 16 \frac{m}{s}$$

$$\mu_n = \frac{U_H \cdot w}{b \cdot B \cdot U} = 2910 \frac{cm^2}{Vs}$$

b) Measuring the Hall voltage as function of magnetic field

As can be seen from the linear regression of a straight line through the origin the Hall voltage U_H is proportional to the magnetic field B:

$U_H \sim B$.

Together with the result of part 1., i.e. $U_H \sim I$, the following relation is found:

$U_H \sim I \cdot B$.

Thus the theoretically derived formula (equation (I)) for the Hall voltage U_H of a strip-shaped conductor of thickness d is confirmed. Form the fit of a straight line to the experimental data of Fig. 5 the Hall coefficient R_H is obtained as follows:

$d = 1{\cdot}10^{-3}$ m

I = 30 mA

A = 0.167 V/T (slope of Fig. 5)

$$R_H = \frac{A \cdot d}{I} = 5.6 \cdot 10^{-3} \frac{m^3}{As}$$

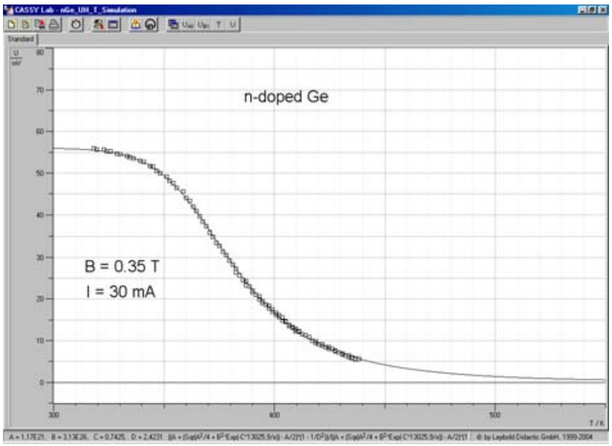

Fig. 7: Fit according equation (XVI) to the experimental data of Fig 6 for B = 0.35 T and I = 30 mA.

A comparison of the Hall coefficient with the Hall coefficient of the metallic conductor silver ($R_H = 8.9{\cdot}10^{-11} mC^{-1}$ experiment P7.2.1.1) shows that the material dependent factor is about 10^7 larger.

c) Measuring the Hall voltage as function of temperature

Using equations (XI) and (XV) together with the relations $n = n_E + n_S$ and $p = p_E$ the Hall voltage U_H can be expressed as follows:

U_H =((A+(Sqr(A^2/4+B^2*Exp(-C*13025.9/x))-A/2)*(1-1/D^2))/((A+(Sqr(A^2/4+B^2*Exp(-C*13025.9/x))-A/2)*(1+1/D))^2)*6.554*10^22) (XVI)

Thus the temperature behavior of the Hall voltage U_H can be simulated with the following fit parameters (For the Fit use key Alt F):

$A = 1.17{\cdot}10^{21}\ m^{-3}$

$B = N_0 = 3.13{\cdot}10^{26}\ m^{-3}$

$C = E_g = 0.74$ eV

$D = \mu_n/\mu_p = 2.4$

The result of the fit is shown in Fig. 7.

The temperature dependency of the Hall voltage U_H probes the transition from a charge transport due to "dopant electrons" to bipolar a charge transport of electron and holes. At room temperatures the observed behavior of U_H is due to electrons created by the donor atoms in the germanium lattice. Increasing the temperature, the charge transport is more and more due to thermally activated electrons and "vacancies" left in the valence band. In contrast to experiment P7.2.1.4 of p-doped germanium no sign change of the Hall voltage U_H is observed as the charge transport is always predominately due to electrons. The drift velocity and thus the mobility of the electrons in the conduction band is larger as the drift velocity and mobility of the holes in the valence band:

$$\mu_n \approx 2.4 \cdot \mu_p$$

For higher temperatures, the charge transport is predominately due to the intrinsic charge carriers, i.e. the electrons and holes. In this temperature region the charge density of

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holes and electrons are approximately the same. Thus the Hall voltage U_H decreases to zero for increasing temperature due to the equal but opposite Hall voltages of the electrons and holes (Fig. 7). For that reason no Hall Effect can be observed in pure semiconductors (intrinsic charge carriers only).

The simplified model neglects the quantum mechanical corrections due to the band theory. Especially, the effective state density N_0 which is given as the product of the effective state densities of the conduction band N_C and valence band. N_V is not constant as assumed in equation (XIV): N_0 has to be replaced by the product of the effective state densities of the conduction band N_C and valence band N_V:

$$N_0 = N_C \cdot N_V \propto T^{\frac{3}{2}} \qquad \text{(XII)}$$

Supplementary information

The Hall effect was discovered in 1879. Although the Hall effect is present in all conducting materials it remained a laboratory curiosity until the later half of 2000 century. With the advent of semiconductor technology and development of various III- and V-compounds it has become possible to produce Hall voltages several orders of magnitude larger than with earlier materials. In technical applications the Hall effect of semiconductors is especially used in magnetic measurement probes.

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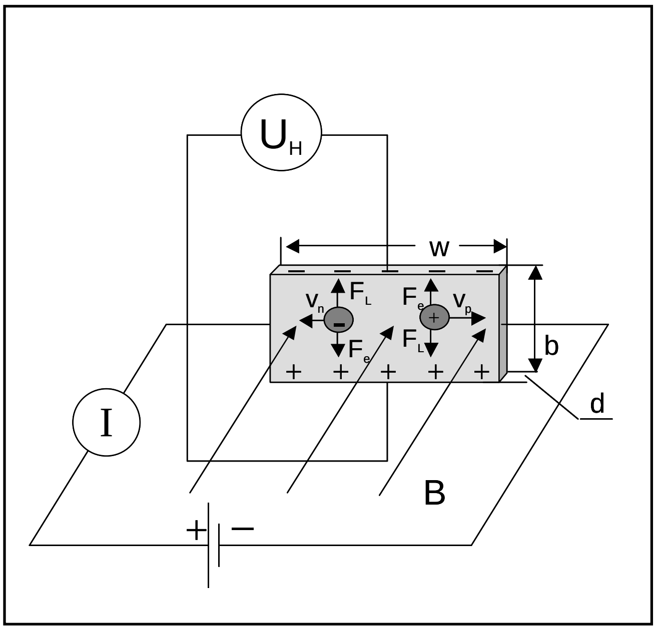
Determining the density and mobility of charge carriers in p-germanium

Objects of the experiment

- Measuring of the Hall voltage as function of the current at a constant magnetic field: determination of the density and mobility of charge carriers.
- Measuring of the Hall voltage for as function of the magnetic field at a constant current:: determination of the Hall coefficient.
- Measuring of the Hall voltage as function of temperature: investigation of the transition from extrinsic to intrinsic conductivity.

Fig. 1: Hall effect in a rectangular sample of thickness d, height b and length w: At equilibrium conditions the Lorentz force F_L acting on the moving charge carriers is balanced by the electrical force F_e which is due to the electric field of the Hall effect.



Principles

The Hall effect is an important experimental method of investigation to determine the microscopic parameters of the charge transport in metals or doped semiconductors.

To investigate the Hall effect in this experiment a rectangular strip of p-doped germanium is placed in a uniform magnetic field B according Fig. 1. If a current I flows through the rectangular shaped sample an electrical voltage (Hall voltage) is set up perpendicular to the magnetic field B and the current I due to the Hall effect:

$$U_H = R_H \cdot \frac{I \cdot B}{d} \qquad \text{(I)}$$

R_H is the Hall coefficient which depends on the material and the temperature. At equilibrium conditions (Fig. 1) for weak magnetic fields the Hall coefficient R_H can be expressed as function of the charge density (carrier concentration) and the mobility of electrons and holes:

$$R_H = \frac{1}{e_0} \cdot \frac{p \cdot \mu_p^2 - n \cdot \mu_n^2}{(p \cdot \mu_p + n \cdot \mu_n)^2} \qquad \text{(II)}$$

$e_0 = 1.602 \cdot 10^{-19}$ As (elementary charge)

$p = p_E + p_S$ (total density of holes)

p_E: density of holes (intrinsic conduction)

p_S: density of holes (hole conduction due to p-doping)

$n = n_E$:density of electrons (intrinsic conduction)

μ_p:mobility of holes

μ_n mobility of electrons

From equation (II) follows: The polarity of predominant charge carriers can be determined from the Hall coefficient R_H if the directions of the current I and magnetic field B are known. The thinner the conducting strip the higher the Hall voltage.

The doping of group III elements like e.g. B, Al, In or Ga into the crystal lattice of germanium creates positive charged holes in the valence band (Fig. 2).

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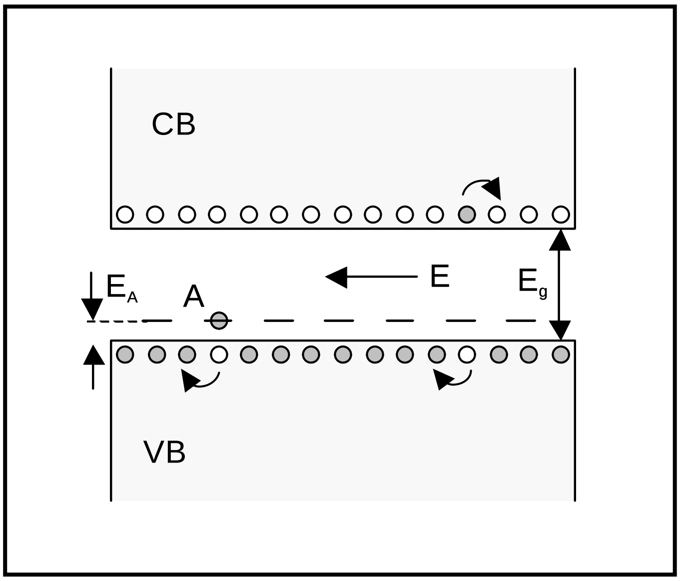

Fig. 2: Simplified diagram of extrinsic (left) and intrinsic conduction (right) under influence of an electric field E: Incorporating of dopants (acceptors A) into the crystal lattice creates positive charge carriers called holes in the valence band (VB). With increasing temperature the thermal energy of valence electrons increases allowing them to breach the energy gap E_g into the conduction band (CB) leaving a vacancy called hole in the VB.

Their activation energy E_A of about 0.01 eV is significantly smaller than the activation energy E_g (band gap) to generate electrons and holes by thermal activation (intrinsic charge carriers). At room temperatures in p-doped germanium the density of holes p_S can predominate the density of intrinsic charge carriers (p_E and n_E). In this case where the charge transport is predominately due to holes from the dopants ($n = n_E = p_E \approx 0$). The density of p_S can be determined by measuring the Hall voltage U_H as function of the current I. With equation (I) and (II) follows:

$$p_S = \frac{B}{e_0 \cdot d} \cdot \frac{I}{U_H} \qquad \text{(III)}$$

The mobility is a measure of the interaction between the charge carriers and the crystal lattice. The mobility is defined as (in case p-doped germanium it is the mobility μ_P of the holes created by the dopants, i.e. acceptors):

$$\mu_P = \frac{v_p}{E} \qquad \text{(IV)}$$

v_p: drift velocity

E: electric field due to the voltage drop

The electric filed E can be determined by the voltage drop U and the length w of the p-doped germanium strip:

$$E = \frac{U}{w} \qquad \text{(V)}$$

The drift velocity v_p can be determined from the equilibrium condition, where the Lorentz force compensates the electrical force which is due to the Hall field (Fig. 1)

$$e_0 \cdot v_d \cdot B = e_0 \cdot E_H \qquad \text{(VI)}$$

which can be expressed using the relation $E_H = b \cdot U_H$ as

$$v_d = \frac{U_H}{b \cdot B} \qquad \text{(VII)}$$

Substituting equation (V) and (VII) in equation (IV) the mobility μ_p of holes can be estimated at room temperatures as follows:

$$\mu_p = \frac{U_H \cdot w}{b \cdot B \cdot U} \qquad \text{(VIII)}$$

The current I in a semiconductor crystal is made up of both hole currents and electron currents (Fig. 1):

$$I = b \cdot d\,(n_p \cdot \mu_p + n_n \cdot \mu_n) \qquad \text{(IX)}$$

The carrier density depends on the dopant concentration and the temperature. Three different regions can be distinguished for p-doped germanium: At very low temperatures the excitation from electrons of the valence band into the acceptor levels is the only source of charge carriers. The density of holes p_S increases with temperature. It follows a region where the density p_S is independent of temperature as all acceptor levels are occupied (extrinsic conductivity). In this regime the charge transport due to intrinsic charge carriers can be neglected. A further increase in temperature leads to a direct thermal excitation of electrons from the valence band into the conduction band. The charge transport increases due to intrinsic conductivity and finally predominates (Fig. 2). These transition from pure extrinsic conduction to a predominately intrinsic conduction can be observed by measuring the Hall voltage U_H as function of the temperature.

To describe the Hall voltage as function of temperature U_H based on a simple theory equation (I) and (II) have to be extended in the following way:

It is assumed that the mobility of electrons and holes are different. Introducing the ratio of the mobility

$$k = \frac{\mu_n}{\mu_p} \qquad \text{(X)}$$

equation (II) can be rewritten as follows:

$$R_H = \frac{1}{e_0} \cdot \frac{p - n \cdot k^2}{(p + n \cdot k)^2} \qquad \text{(XI)}$$

For undoped semiconductors the temperature dependency of the charge carriers can be assumed as

$$n = n_0 \cdot e^{-\frac{E_g}{2 \cdot k_B \cdot T}}$$

$$p = p_0 \cdot e^{-\frac{E_g}{2 \cdot k_B \cdot T}} \qquad \text{(XII)}$$

$k_B = 1.36\ 10^{-23}$ J/K: Boltzmann constant

The product of the densities p and n is temperature dependent:

$$n \cdot p = n_E \cdot (p_E + p_S) = \eta^2 \qquad \text{(XIII)}$$

where the effective state density η is approximated as

$$\eta^2 = N_0 \cdot e^{-\frac{E_g}{k_B \cdot T}} \qquad \text{(XIV)}$$

In the extrinsic conductivity regime the density p_S of holes can be determined according equation (III). For the intrinsic charge carriers $p_E = n_E$ which leads to a quadratic equation for p_E with the solution:

$$p_E = -\frac{P_S}{2} + \sqrt{\frac{P_S^2}{4} + \eta^2} \qquad \text{(XV)}$$

With equations (XI) and (XV) together with the relations $p = p_E + p_S$ and $n = n_E$ the temperature dependency of Hall voltage U_H can be simulated. Using for E_g = 0.7 eV the result of experiment P7.2.1.5 as estimate value for the simulation only two unknown parameters N_0 and k are left.

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1 DC Power Supply 0...16 V, 0...5 A..................... 521 545
1 DC power supply 521 541

1 U-core with yoke... 562 11
1 Pair of bored pole pieces................................... 560 31
2 Coil with 250 turns.. 562 13
1 Stand rod, 25 cm 300 41
1 Leybold Multi clamp... 301 01
1 Stand base, V-shape, 20 cm 300 02
7 Pair of cables, 1 m, red and blue....................... 501 46

additionally required:

PC with Windows 95/98/NT or higher

Setup

Mounting and connecting the plug-in board:

- Insert the plug-in board with the p-doped Ge crystal into the DIN socket on the base unit for Hall effect until the pins engage in the holes.
- Carefully insert the plug-in board with DIN plug into the DIN socket on Insert the base unit with rod into the hole of the U-core all the way to the stop; make sure that the plug-in board is seated parallel to the U-core (see instruction sheet base unit Hall effect 586 850).
- Carefully attach the pair of bored pole pieces with additional pole piece, and slide the additional pole piece as far as the spacers of the plug-in boards (make sure that the plug-in board is not bent).
- Turn the current limiter of the current-controlled power supply to the left stop, and connect the power supply.

Safety notes

The n-doped Ge crystal is extremely fragile:

- Handle the plug-in board carefully and do not subject it to mechanical shocks or loads.

Due to its high specific resistance, the p-doped Ge crystal warms up even if only the cross-current is applied:

- Do not exceed the maximum cross-current I = 33 mA.
- Turn the control knob for the cross-current on the base unit for Hall effect to the left stop.

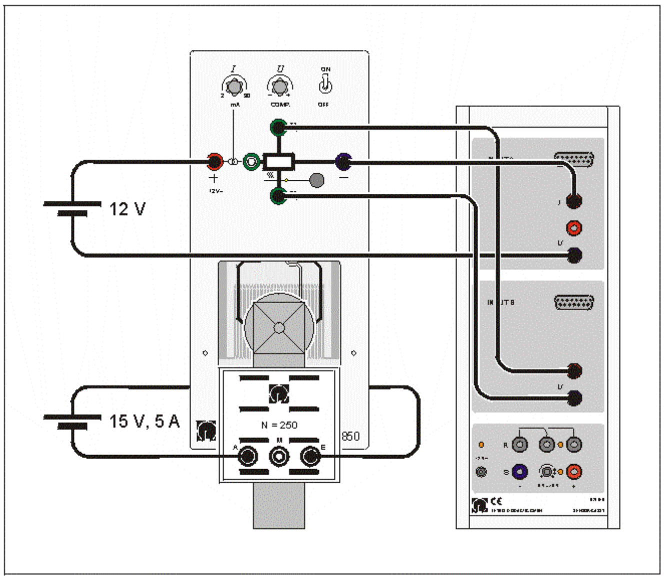

Fig. 2: Experimental setup (wiring diagram) for measuring the Hall voltage as function of the current I.

Measuring the magnetic field:

- The B-probe is fixed by the Stand rod to the V-shaped Stand base.
- Before the measuring the magnetic induction of the field B place the B-probe carefully in the gap (see instruction sheet base unit Hall effect 586 850) after the apparatus is adjusted.
- For the measurement connect B-probe to the Sensor CASSY using the extension cable.

Compensation of the Hall voltage:

- Before performing a measurement with a constant current I the Hall voltage have to be compensated for B = 0 T:
- 1. For measuring the current I connect the cables to the Input A of the Sensor CASSY (Fig. 3, see also instruction sheet base unit Hall effect 586 850).
- 2. For measuring the Hall voltage U_H connect the cables to the Input B of the Sensor CASSY (Fig. 3 see also instruction sheet base unit Hall effect 586 850).
- 3. Set the cross-current I to the maximum value (see instruction manual for n-doped Ge crystal 586 852), switch on the compensation and zero the Hall voltage U_H using the compensation knob.

Measuring the voltage drop:

- For measuring the voltage drop U connect the cables to the Input B of Sensor CASSY (see instruction sheet base unit Hall effect 586 850 measure the conductivity as function of temperature).
- Connect the cables to the Input A of the Sensor CASSY to measure the current I (see instruction sheet base unit Hall effect 586 850).
- Set the current I to the maximum value and measure the voltage drop U.

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Measuring the temperature:

- For measuring the temperature ϑ connect the output signal of the heater to Input A of the Sensor CASSY (see instruction sheet base unit Hall effect 586 850 and Physics Leaflets P7.2.1.5.)

Carrying out the experiment

a) Measuring the Hall voltage as function of current

- First compensate the Hall voltage (see above).
- Set the magnetic field B to a desired value and measure the magnetic flux density B (see above).
- Set the current to the maximum value and measure the voltage drop U.
- Measure the Hall voltage U_H (Input B on Sensor CASSY) as function of the current I (Input A on Sensor CASSY).
- After connecting the cables set the parameters with .
- For measuring use the button or F9 in manual measuring mode.
- Safe your measurement .

b) Measuring the Hall voltage as function of magnetic field

- First compensate the Hall voltage (see above).
- Set the current I to a desired value.
- Measure the Hall voltage U_H (Input B on Sensor CASSY) as function of the magnetic field B (Input A on Sensor CASSY).
- After connecting the cables set the parameters with .
- For measuring use the button or F9 in manual measuring mode.
- Safe your measurement .

c) Measuring the Hall voltage as function of temperature

- First compensate the Hall voltage U_H (see above) and set the current I to a desired value.
- Set the magnetic field B to a desired value (see above).
- Measure the Hall voltage U_H (Input B on Sensor CASSY) as function of the Temperature ϑ (Input A on Sensor CASSY, see above).

Measuring example

a) Measuring the Hall voltage as function of current

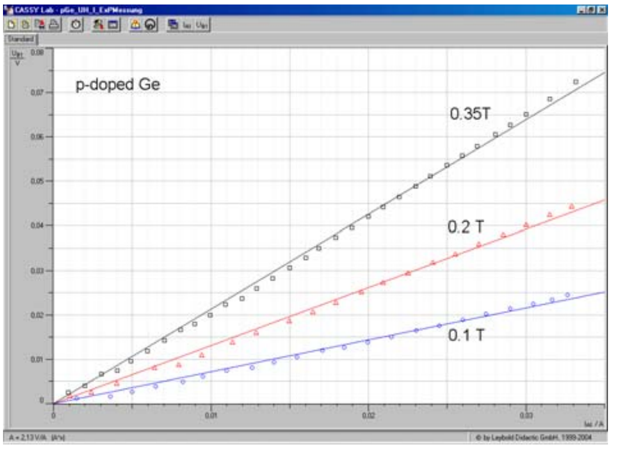

Fig. 4: Hall voltage U_H as function of the current I for different magnetic fields. The straight lines correspond to a fit according equation (I).

current: I = 30 mA

voltage drop: U = 1,4 V.

b) Measuring the Hall voltage as function of magnetic field

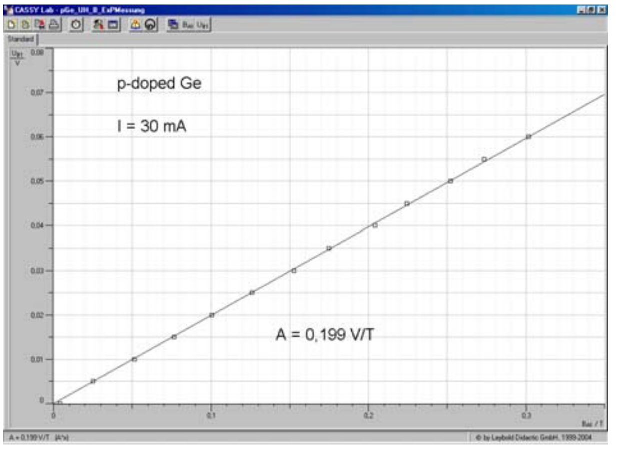

Fig. 5: Hall voltage U_H as function of the magnetic field B for I = 30 mA.. The straight line with slope A corresponds to a fit according equation (I).

c) Measuring the Hall voltage as function of temperature

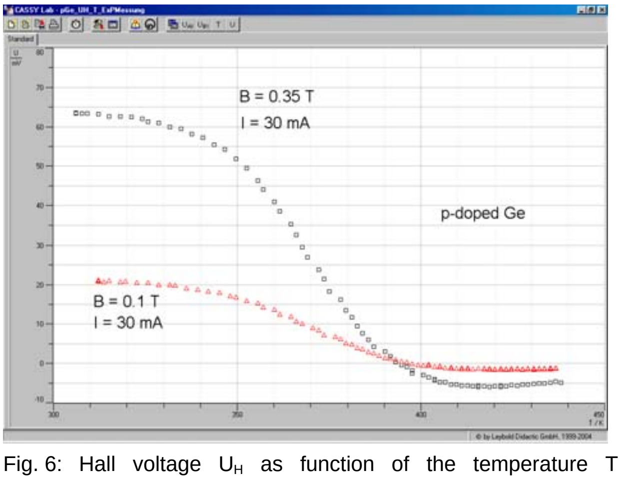

Fig. 6: Hall voltage U_H as function of the temperature T for I = 30 mA and different magnetic fields B.

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Evaluation and results

a) Measuring the Hall voltage as function of current

For the measurement with e. g. B = 0,35 T and I = 30 mA in Fig. 4 the slope

$$A = \frac{R_H \cdot B}{d} = 2.13\,\frac{V}{A}$$

is obtained by the fitting a straight line through the origin (right mouse click in the diagram and "fit function"). With the linear regression result and equation (III) the density p_S of holes in the extrinsic conducting regime can determined as follows:

$d = 1{\cdot}10^{-3}$ m

B = 0.35 T

$$p_S = \frac{B}{e_0 \cdot d \cdot A} = 1.1 \cdot 10^{21}\,\frac{1}{m^3}$$

With the experimental results at room temperature

U = 1.4 V

B = 0.35 T

U_H = 72 mV

and the dimensions of the p-doped germanium strip

b = 10 mm

w = 20 mm

the drift velocity v_p (equation (VII)) and the mobility μ_p (equation (VIII)) of the charge carriers in the extrinsic region can be estimated:

$$v_p = \frac{U_H}{b \cdot B} = 21\frac{m}{s}$$

$$\mu_p = \frac{U_H \cdot w}{b \cdot B \cdot U} = 2940\,\frac{cm^2}{Vs}$$

b) Measuring the Hall voltage as function of magnetic field

As can be seen from the linear regression of a straight line through the origin the Hall voltage U_H is proportional to the magnetic field B:

$U_H \sim B$.

Together with the result of part 1., i.e. $U_H \sim I$, the following relation is found:

$U_H \sim I \cdot B$.

Thus the theoretically derived formula (equation (I)) for the Hall voltage U_H of a strip-shaped conductor of thickness d is confirmed. Form the fit of a straight line to the experimental data of Fig. 5 the Hall coefficient R_H is obtained as follows:

$d = 1{\cdot}10^{-3}$ m

I = 30 mA

A = 0.199 V/T (slope of Fig. 5)

$$R_H = \frac{A \cdot d}{I} = 6.6 \cdot 10^{-3}\,\frac{m^3}{As}$$

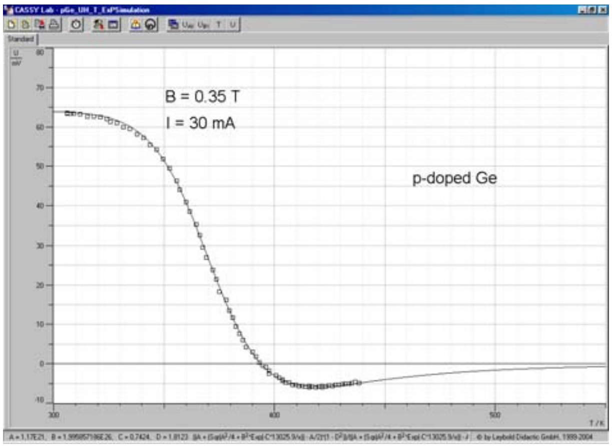

Fig. 7: Fit according equation (XVI) to the experimental data of Fig 6 for B = 0.35 T and I = 30 mA.

A comparison with e. g. the Hall coefficient of the metallic conductor silver ($R_H = 8.9{\cdot}10^{-11} mC^{-1}$ experiment P7.2.1.1) shows that the Hall coefficient is about 10^7 larger for semi-conductors.

c) Measuring the Hall voltage as function of temperature

Using equations (XI) and (XV) together with the relations $p = p_E + p_S$ and $n = n_E$ the Hall voltage U_H can be expressed as follows:

U_H = ((A+(Sqr(A^2/4+B^2*Exp(-C*13025.9/x))-A/2)*(1-D^2))/((A+(Sqr(A^2/4+B^2*Exp(-C*13025.9/x))-A/2)*(1+D))^2)*7.49*10^22)

(XVI)

Using equation (XVI) the temperature behavior of the Hall voltage U_H can be simulated with the following fit parameters (For performing a Fit with CASSY Lab use key Alt F):

$A = 1.17{\cdot}10^{21}\ m^{-3}$

$B = N_0 = 1.99{\cdot}10^{26}\ m^{-3}$

$C = E_g = 0.74$ eV

$D = \mu_n/\mu_p = 1.81$

The result of the fit is shown in Fig. 7.

The temperature dependency of the Hall voltage U_H probes the transition from a charge transport due to holes to a bipolar charge transport of electrons and holes. At room temperatures the observed behavior of U_H is due to holes created by the acceptor atoms in the germanium lattice. Increasing the temperature, the charge transport is more and more due to thermally activated electrons and "vacancies" left in the valence band. When the number of the "faster" electrons exceeds the number of holes the Hall voltage U_H becomes negative. According equation (II) a sign change of U_H takes place when

$$p \cdot \mu_p^2 = n \cdot \mu_n^2 .$$

The negative temperature range of the Hall voltage is determined by the electrons. Their drift velocity and thus their mobility is larger as the drift velocity and mobility of the holes, respectively.

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$\mu_n \approx 2 \cdot \mu_p$

At high temperatures the charge density of holes and electrons are approximately the same. The Hall voltage U_H approaches finally zero due to the equal but opposite electrical fields of the electrons and holes (Fig. 7). For that reason no Hall Effect can be observed in pure semiconductors (intrinsic charge carriers only).

The simplified model neglects corrections of the quantum theory, i.e. band structure and effective mass. Especially, the effective state density N_0 is not constant as assumed in equation (XIV). N_0 has to be replaced by the product of the effective state densities of the conduction band N_C and valence band N_V:

$$N_0 = N_C \cdot N_V \propto T^{\frac{3}{2}} \qquad \text{(XII)}$$

Supplementary information

The Hall effect was discovered in 1879. Although the Hall effect is present in all conducting materials it remained a laboratory curiosity until the later half of 2000 century. With the advent of semiconductor technology and development of various III- and V-compounds it has become possible to produce Hall voltages several orders of magnitude larger than with earlier materials. In technical applications the Hall effect of semiconductors is especially used in magnetic measurement probes.

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Determining the band gap of germanium

Objects of the experiments

- Determining the voltage drop at an undoped Ge crystal as a function of the temperature when the current through the crystal is constant, and calculating the conductivity σ.
- Determining the band gap E_g of germanium.

Principles

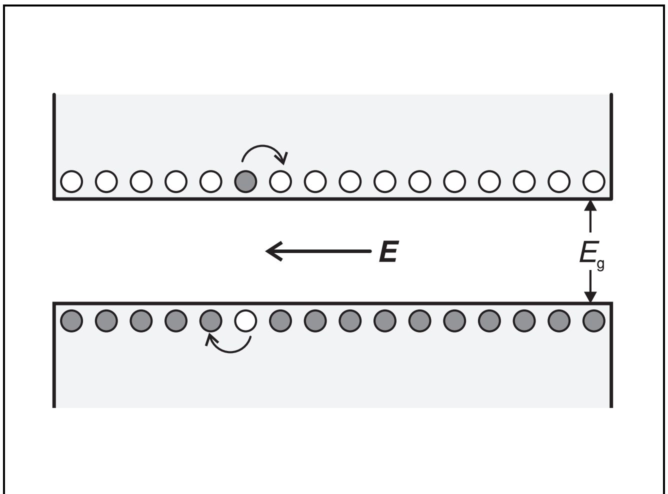
For the current density j in a body under the influence of an electric field E the Ohm law states

$$j = \sigma \times E \qquad \text{(I).}$$

The proportionality factor σ is called electric conductivity. Since this quantity strongly depends on the material, it is common to classify materials with regard to their conductivity. Semiconductors, for example, are solids that do not conduct electric currents at low temperatures, but show a measurable conductivity at higher temperatures. The reason for this temperature dependence is the specific band structure of the electronic energy levels of a semiconductor.

The valence band, i.e. the highest band that is completely or partially populated in the ground state, and the conduction band, i.e. the next unpopulated band, are separated by a band gap E_g (Ge: $E_g \approx$ 0,7 eV). The region between the two bands is not populated by electrons in an undoped, pure semiconductor and is called the “forbidden zone“. At higher temperatures, more and more electrons are thermally activated from the valence band into the conduction band. They leave “holes” in the valence band which move like positive charged particles, thus contributing to the current density j as do the electrons (see Fig. 1).

Fig. 1 Simplified diagram of intrinsic conduction: a semiconductor with an electron in the conduction band and a hole in the valence band under the influence of an electric field E.



The conduction which is made possible by the excitation of electrons from the valence band into the conduction band is called intrinsic conduction. Since under conditions of thermal equilibrium the numbers of holes in the valence band and of electrons in the conduction band are equal, the current density in the case of intrinsic conduction can be written in the form

$$j_i = (-e) \times n_i \times v_n + e \times n_i \times v_p \qquad \text{(II)}$$

e: elementary charge,
n_i: concentration of electrons or holes respectively.

The average drift velocities v_n and v_p of the electrons and the holes are proportional to the field strength E. With

$$v_n = -\mu_n \times E \text{ and } v_p = \mu_p \times E \qquad \text{(III),}$$

where the mobilities μ_n and μ_p are chosen to be positive quantities,

$$j_i = e \times n_i \times (\mu_n + \mu_p) \times E \qquad \text{(IV)}$$

is obtained. Comparison with (I) leads to

$$\sigma_i = e \times n_i \times (\mu_n + \mu_p) \qquad \text{(V)}$$

for the conductivity. Apart from the elementary charge e, all quantities in (V) depend on the temperature T. The concentration of intrinsic conduction n_i is

$$n_i \times = (N \times P)^{\frac{1}{2}} \times e^{-\frac{E_g}{2 \times kT}} \qquad \text{(VI),}$$

k: Boltzmann constant,
E_g: band gap of the semiconductor.

Apparatus

1 Ge undoped on plug-in board	586 851
1 base unit for Hall effect	586 850
1 sensor CASSY	524010
1 CASSY Lab	524 200
1 current-controlled power supply, 15 V–, 3 A, for example	521 50
1 power supply, 12 V–, 50 mA for example	521 54
1 stand base, V-shape, 20 cm	300 02
connection leads	

$$N = 2 \cdot \left(\frac{2\pi \cdot m_n \cdot kT}{h^2}\right)^{\frac{3}{2}} \text{ and } P = 2 \cdot \left(\frac{2\pi \cdot m_p \cdot kT}{h^2}\right)^{\frac{3}{2}} \qquad \text{(VII)},$$

h: Planck constant,
m_n: effective electron mass,
m_p: effective hole mass,

are the effective state densities in the conduction band and in the valence band. The mobilities μ_n and μ_p also depend on the temperature. At low temperatures, the proportionality $\mu \propto T^{-\frac{3}{2}}$ holds roughly, as does the proportionality $\mu \propto T^{-\frac{3}{2}}$ at high temperatures.

Because of the predominance of the exponential function (see Eq. (VI)), conductivity is well approximated and represented by

$$\sigma_i = \sigma_0 \cdot e^{-\frac{E_g}{2 \cdot kT}} \qquad \text{(VIII)}$$

or

$$\ln \sigma_i = \ln \sigma_0 - \frac{E_g}{2 \cdot kT} \qquad \text{(IX)}.$$

With the object of confirming Eq. (VIII) and determining the band gap E_g, the conductivity of undoped germanium is determined as a function of the temperature *T* in the experiment. At a constant current

$$I = j \cdot b \cdot c \qquad \text{(X)},$$

b: breadth of the crystal, *c:* thickness of the crystal,

the voltage drop

$$U = E \cdot a \qquad \text{(XI)},$$

a: length of the crystal,

is measured at an undoped Ge crystal.

Because of (I), (X), and (XI) the conductivity

$$\sigma = \frac{a}{b \cdot c} \cdot \frac{I}{U} \qquad \text{(XII)}.$$

Setup

The experimental setup is illustrated in Fig. 2.

Mounting and connecting the plug-in board:

Notes:

The Ge crystal is extremely fragile:
Handle the plug-in board carefully and do not subject it to mechanical shocks or loads.

Due to its high specific resistance, the Ge crystal warms up even if only the cross-current is applied:
Do not exceed the maximum cross-current I = 4 mA.
Turn the control knob for the cross-current on the base unit for Hall effect to the left stop.

– Insert the plug-in board with the Ge crystal into the DIN socket on the base unit for Hall effect until the pins engage in the holes.
– Turn the current limiter of the current-controlled power supply to the left stop, and connect the power supply to the input for the heating and electronics of the base unit for Hall effect.
– Switch the current-controlled power supply on, set the voltage limiter to 15 V and the current limiter to 3 A (for this, short-circuit the output of the power supply temporarily).
– Turn the control knob for the cross-current on the base unit for Hall effect to the left stop, and supply the current source by connecting the other power supply. Switch this power supply on and set its output voltage to 12 V.

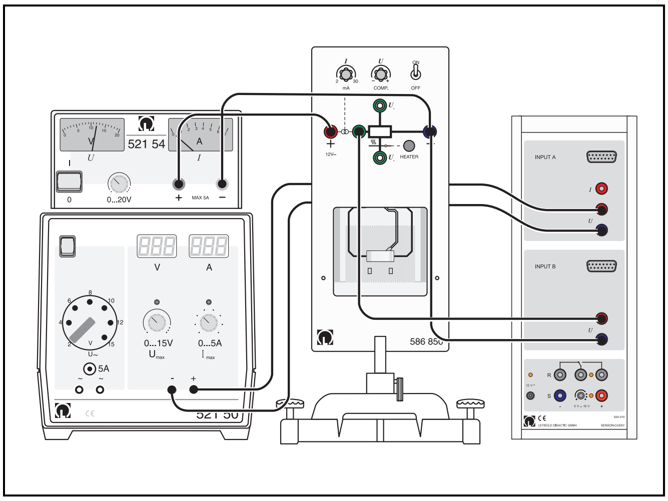


Fig. 2 Experimental setup for the determination of the band gap of germanium

Preparation for data logging:

– Connect the output for the temperature measurement to input A and the output for the voltage drop to input B of sensor CASSY.
– Connect sensor CASSY to the serial interface of the PC (usually COM1 or COM2) with the nine-pole V24 cable.
– If necessary, install the “CASSY Lab” program under Windows 95/98/NT and choose the desired language.
– Get the “CASSY Lab” program started and check whether connection of the CASSY sensor is correct.

– Erase existing data from previous measurements with the button or the key F4.
– Open the pop-up window “Settings” with the button or the key F5, and click “Update Setup“.
– Click channel A and measurand “UA1“, zero “Left“, and choose the measuring range “0 … 3 V“. Perform the same settings for channel B.
– In the register “Display” set:

x-axis	UA1	x
y-axis	UB1	y

– Click “Display Measuring Parameters” and choose the measuring parameters “Automatic Recording” and “Meas. Interv.: 2 s”.

Carrying out the experiment

Press the key HEATER on the base unit for Hall effect, and start recording of the measuring values with the key F9 or the button .

– In the window “Voltage UA1” check if the voltage UA1, which is proportional to the temperature of the crystal, increases.

As soon as the voltage UB1 has fallen below 1 V:

– Click the window “Voltage UB1” with the right mouse button, and change the measuring range to “0 … 1 V“.

As soon as the voltage UB1 has fallen below 0.3 V:

– change to the measuring range “0 V.. 0.3 V“.

When the LED on the base unit for Hall effect goes out:

– Stop the record of the measuring values with the key F9 or the button .

For the further evaluation:

– Define the following as new quantities in the register “Parameter/Formula/FFT”:

Quantity	Temperature	Conductivity
Formula	UA1*100+273,15	4/UB1
Symbol	T	&s
Unit	K	1/&Wm
from	290	0
to	440	200
Decimals	1	2

– In the register “Display” choose as new representation:

x-axis	T	1/x
y-axis	σ	log y

– After clicking the graphics window with the right mouse button, activate the item “Fit function” → “Best –fit straight line“, and mark the desired range of adjustment with the left mouse button.

Measuring example

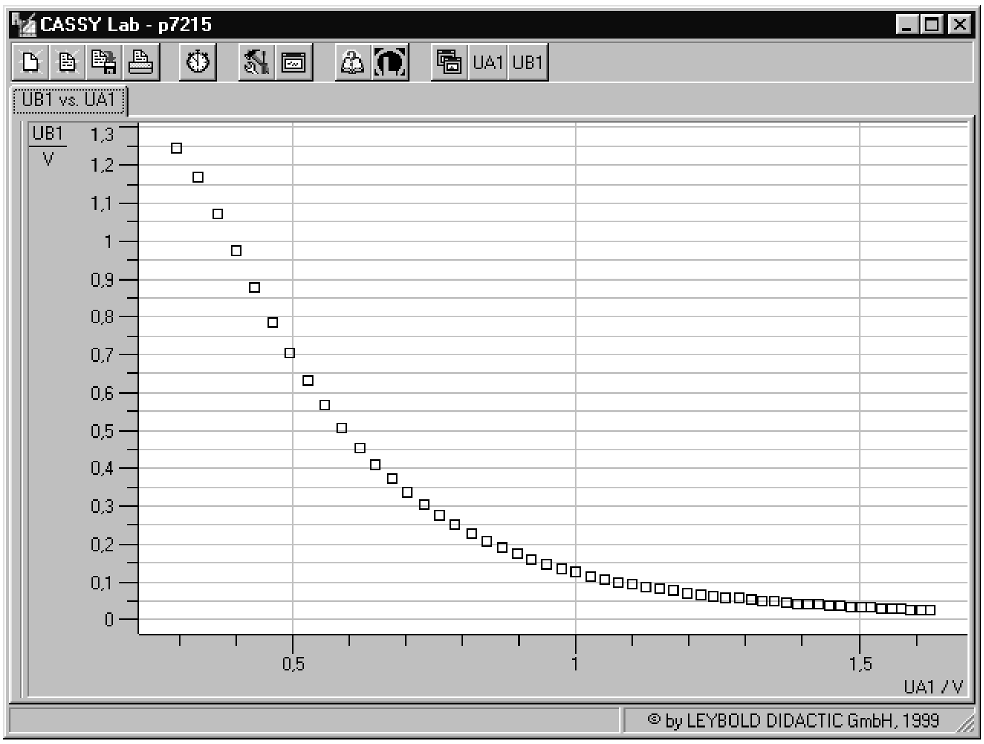


Fig. 3 Graph of the measuring values in the form UB1 = *f*(UA1)
UA1: voltage at the output for temperature measurement
UB1: voltage drop at the Ge crystal at a cross-current of 2 mA

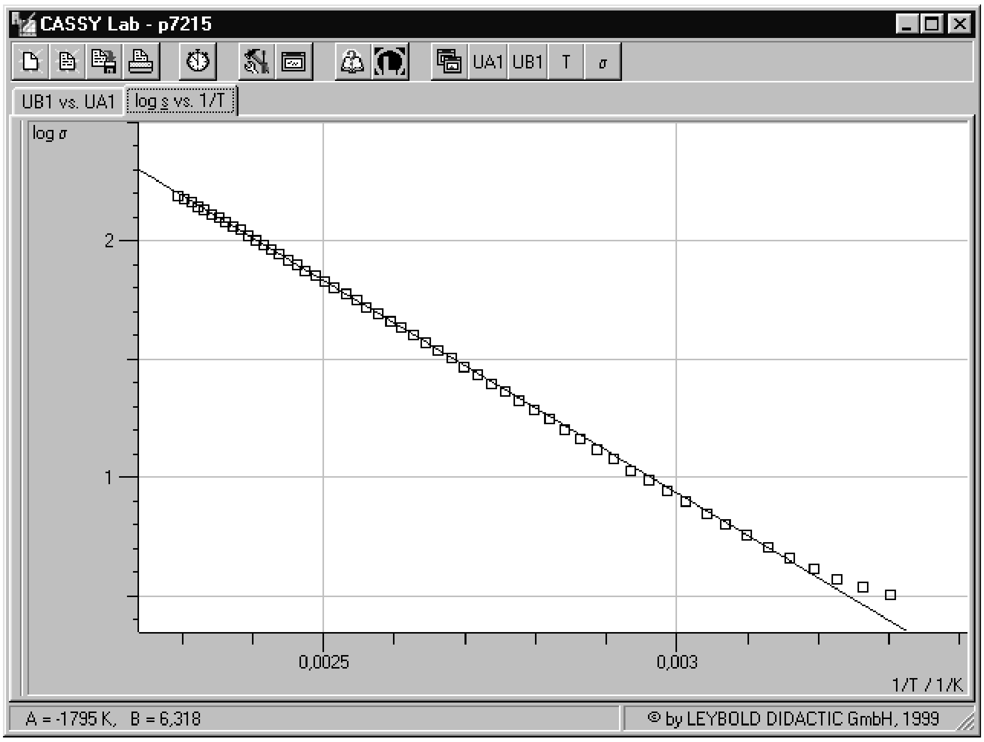


Fig. 4 Graph of the measuring values in the form $\log\sigma = f\left(\frac{1}{T}\right)$

$\sigma = \frac{2\text{mA}}{\text{UB1}} \times \frac{20\text{mm}}{10\text{mm} \times 1\text{mm}}$ (conductivity, cf. (XII))

$T = 100\text{K} \times \frac{\text{UA1}}{\text{V}} + 273.15\text{ K}$ (temperature)

Evaluation and results

In the graph $\log\sigma = f(\frac{1}{T})$, the data points are well approximated on a straight line with the slope A = –1795 K (see Fig. 4).

According to (IX), the slope of the straight line is

$$A = -\frac{1}{\ln 10} \times \frac{E_g}{2 \times k} \qquad \text{(XIII)},$$

with $k = 1.3807 \times 10^{-23}$ J K^{-1}.

From this the band gap

$E_g = 1.149 \times 10^{-19}$ J = 0,71 eV

Is obtained.

Values quoted in the literature:

E_g(0 K) = 0.74 eV, E_g(300 K) = 0.67 eV

Supplementary information

The temperature measurement is distorted by hysteresis in the lower temperature range. Therefore, the corresponding data points in Fig. 4 are systematically higher than the straight line drawn in the graph.

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